Logarithme en base a

- Propriétés
- $\forall\, x \in \mathbb{R}_0^+ : y = \log_a x \iff x = a^y$

$\forall\, x \in \mathbb{R} : \log_a(a^x) = x$

$\forall\, x \in \mathbb{R}_0^+ : a^{\log_a x} = x$

- Propriétés

$\forall\, x,\, y \in \mathbb{R}_0^+$

- $\log_a 1 = 0$

$\log_a 1 = \log_a a^0 = 0$

- $\log_a a = 1$

$\log_a a = \log_a a^1 = 1$

- $\log_a(x.y) = \log_a x + \log_a y$

on pose $u = \log_a x$ et $v = \log_a y$
on a alors $x = a^u$ et $y = a^v$

$\log_a(x.y) = \log_a(a^u.a^v) = \log_a a^{u+v} = u + v = \log_a x + \log_a y$

- $\log_a\left(\frac{1}{x}\right) = -\log_a x$

on pose $u = \log_a x$
on a alors $x = a^u$

$\log_a\left(\frac{1}{x}\right) = \log_a\left(\frac{1}{a^u}\right) = \log_a a^{-u} = -u = -\log_a x$

- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a\left(x.\frac{1}{y}\right) = \log_a x + \log_a \frac{1}{y} = \log_a x - \log_a y$

- $\log_a x^n = n\,.\log_a x$

on pose $u = \log_a x$
on a alors $x = a^u$

$\log_a x^n = \log_a(a^u)^n = \log_a a^{n\cdot u} = n\,.u = n\,.\log_a x$

- $\log_a x = \dfrac{\log_b x}{\log_b a}$

on pose $u = \log_a x$

on a alors $x = a^u$

$\log_b x = \log_b a^u = u \,.\, \log_b a = \log_a x \,.\, \log_b a$

$\log_b x = \log_a x \,.\, \log_b a$

$\log_a x = \dfrac{\log_b x}{\log_b a}$